ISSN 2229-5518

The Skin Friction Analysis of Exponentially Accelerated Vertical Plate with Variable Mass Diffusion

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ABSTRACT

In this paper the unsteady laminar free-convection flow of a viscous incompressible fluid, past an exponentially accelerated infinite vertical plate with variable mass diffusion is considered. The Laplace transform method is used to obtain the expression for skin-friction, Nusselt number and Sherwood number. Numerical computations are carried out for different physical parameters like Prandt number, thermal Grashof number, mass Grashof number, Schmidt number and time. It is observed that skin-friction decreases with increasing radiation parameter and time.

Key words: exponential, radiation, skin friction, vertical plate.

INTRODUCTION

Natural convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method by Gupta *et al* (1979). Soundalgekar (1982) studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh (1983). Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar (1984). The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo (1986). Mass transfer effects on the flow past an accelerated infinite vertical plate with variable heat-transfer was studied by Raptis *et al* (1981).Mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion was studied by Basant Kumar Jha *et al* (1991).

The object of the present paper is to study the effects of skin-friction, the rate of heat and mass transfer on an exponentially accelerated infinite vertical plate with uniform mass diffusion in the presence of thermal radiation. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and error functions.

MATHEMATICAL ANALYSIS

Here, an unsteady flow of a viscous incompressible fluid past an exponentially accelerated infinite isothermal vertical plate with uniform mass diffusion, in the presence of thermal radiation is studied. Consider the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞. Here, the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and

ISSN 2229-5518

concentration. At time t'>0, the plate is exponentially accelerated with a velocity $u=u_0 \exp(a't')$ in its own plane and the temperature from the plate is raised to T_w and the concentration level is made to rise linearly with time t. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Then by usual Boussinesqs' approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \qquad \textbf{(1)}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\Pr}\theta \qquad (2)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \qquad (3)$$

subject to the initial and boundary conditions in non-dimensional form

$$\begin{array}{llll} & U=0, & \theta=0, & C=0, \quad for \quad all\, Y,\ t<0 \\ t>0: & U=u_0\exp(at) & \theta=1, & C=t \quad at \quad Y=0 \\ & U=0 & \theta\to 0, & C\to 0 \; as \quad Y\to\infty \end{array} \qquad (4)$$

The non-dimensional quantities introduced in the above equations are defined as

$$U=\frac{u}{u_0},\ t=\frac{t'u_0^2}{\nu},\ Y=\frac{yu_0}{\nu},\ \theta=\frac{T-T_\infty}{T_w-T_\infty},$$

$$Gr=\frac{g\beta\nu(T_w-T_\infty)}{u_0^3},\ C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\ Gc=\frac{\nu g\beta^*(C'_w-C'_\infty)}{u_0^3},$$

$$a=\frac{a'\nu}{u_0^2},$$

$$R=\frac{16a^*\nu^2\sigma T_\infty^3}{ku_0^2},\ Pr=\frac{\mu C_p}{k},\ Sc=\frac{\nu}{D},. \qquad (5)$$

The solutions of equations under the boundary condition have been obtained by Muthucumaraswamy and Visalakshi (2010).

The solution of equation under the boundary condition (4) by applying the Laplace tansformation technique given by

$$\theta=\frac{1}{2}\left[\begin{array}{l}\exp(2\eta\sqrt{Rt})erfc(\eta\sqrt{\Pr}+\sqrt{bt})\\ +\exp(-2\eta\sqrt{Rt})erfc(\eta\sqrt{\Pr}-\sqrt{bt})\end{array}\right] \textbf{(6)}$$

$$C=t\left[\begin{array}{l}(1+2\eta^2 Sc)\,erfc(\eta\sqrt{Sc})\\ -\frac{2\eta}{\sqrt{\pi}}\sqrt{Sc}\exp(-\eta^2 Sc)\end{array}\right] \qquad \textbf{(7)}$$

$$U=\frac{\exp(at)}{2}\left[\begin{array}{l}\exp(2\eta\sqrt{at})erfc(\eta+\sqrt{at})\\ +\exp(-2\eta\sqrt{at})erfc(\eta-\sqrt{at})\end{array}\right]$$

$$-d\exp(ct)\left[\begin{array}{l}\exp(2\eta\sqrt{ct})erfc(\eta\sqrt{\Pr}+\sqrt{ct})\\ +\exp(-2\eta\sqrt{ct})erfc(\eta\sqrt{\Pr}-\sqrt{ct})\end{array}\right]$$

$$-e\left[\begin{array}{l}(3+12\eta^2+4\eta^4)erfc(\eta)-\frac{\eta}{\sqrt{\pi}}(10+4\eta^2)\exp(-\eta^2)\\ -(3+12\eta^2 Sc+4\eta^4(sc)^2)erfc(\eta\sqrt{Sc})\\ -\frac{\eta\sqrt{Sc}}{\sqrt{\pi}}(10+4\eta^2 Sc)\exp(-\eta^2 Sc)\end{array}\right]$$

$$+2\,d\,erfc(\eta) \qquad \textbf{(8)}$$

$$-d\left[\begin{array}{l}\exp(2\eta\sqrt{Rt})erfc(\eta\sqrt{\Pr}+\sqrt{bt})\\ +\exp(-2\eta\sqrt{Rt})erfc(\eta\sqrt{\Pr}-\sqrt{bt})\end{array}\right]$$

$$-d\exp(ct)\left[\begin{array}{l}\exp(-2\eta\sqrt{\Pr(b+c)t})erfc(\eta\sqrt{\Pr}-\sqrt{(b+c)t})\\ +\exp(2\eta\sqrt{\Pr(b+c)t})erfc(\eta\sqrt{\Pr}+\sqrt{(b+c)t})\end{array}\right]$$

Where,

$$b=\frac{R}{Pr},\quad c=\frac{R}{1-Pr},\quad d=\frac{Gr}{2c(1-Pr)},\quad e=\frac{Gct^2}{6(1-Sc)}$$

$\eta = Y/2\sqrt{t}$ and erfc is called complementary error function.

By using equation (8), the skin-friction is given by the following expression

$$\tau=-\left(\frac{dU}{dy}\right)_{y=0}$$

ISSN 2229-5518

$$= -\frac{1}{2\sqrt{t}}\left(\frac{dU}{d\eta}\right)_{\eta=0}$$

$$\frac{e^{at}}{\sqrt{\pi t}} + \sqrt{a}e^{at}erf(\sqrt{at}) + \frac{(4+\sqrt{Sc})}{3\sqrt{\pi}}\frac{Gct^{3/2}}{(1-Sc)} - \frac{Gr}{R\sqrt{\pi t}}$$

$$-\frac{Gr}{R\sqrt{t}}\left[\begin{array}{l} 2e^{ct}\sqrt{\frac{\Pr}{\pi}} + \sqrt{ct}e^{ct}erf(\sqrt{ct}) + \sqrt{Rt}\,erf(\sqrt{bt}) \\ + \sqrt{\frac{\Pr}{\pi}} + e^{ct}\sqrt{\Pr(b+c)t}\,erf(\sqrt{(b+c)t}) \end{array}\right] \quad (9)$$

From the knowledge of temperature field of equation (6), we now study the rate of heat transfer which is expressed in terms of Nusselt number in non- dimensional form as

$$Nu = -\left(\frac{d\theta}{dy}\right)_{y=0}$$

$$= -\frac{1}{2\sqrt{t}}\left(\frac{d\theta}{d\eta}\right)_{\eta=0}$$

$$= \sqrt{\frac{\Pr}{\pi t}} + \sqrt{R}erf(\sqrt{bt}) \quad (10)$$

The numerical values of Nu are entered in Table 3.
From the concentration field, i.e., from equation (7), the rate of mass transfer is expressed in terms of Sherwood number in non dimensional form as given below

$$Sh = -\left(\frac{dC}{dy}\right)_{y=0}$$

$$= -\frac{1}{2\sqrt{t}}\left(\frac{dC}{d\eta}\right)_{\eta=0}$$

$$= 2\sqrt{\frac{tSc}{\pi}} \quad (11)$$

From expression (11), it is understood that rate of mass transfer is directly proportional to square root of time and Schmidt number.
The numerical values are entered in Table 1 & Table 2.

DISCUSSION OF RESULTS

The numerical values of the skin-friction, Nusselt number and Sherwood number are calculated for different physical parameters like thermal Grashof number, mass Grashof number, time in the presence of air (Pr=0.71). The purpose of the calculations given here is to study the effects of the parameters R, t, Gr, Gc and Sc upon the nature of the flow and transport. The solutions are in terms of exponential and error function.

In table 1, the numerical values of the skin friction are computed for Pr=0.71, Sc=0.16, Gr=2 and Gc=5. It is observed from this table, that an increase in the radiation parameter leads to fall in the value of the skin-friction but an increase in the exponential parameter leads to rise in the value of the skin-friction. As time advances the value of the skin- friction decreases.

Table 1
Numerical values of skin-friction when Pr=0.71, Sc=0.16, Gr=2, Gc=5

R	a/t	0.1	0.2	0.3
2	0.1	-12.435733	-26.217266	-52.534629
	0.3	-12.383833	-26.132692	-52.419323
	0.5	-12.323889	- 26.029464	-52.272311
	1.0	-12.145339	-25.698707	-51.773049
3	0.1	-14.363300	-42.616068	-118.938574
	0.3	-14.311400	-42.531494	-118.823268
	0.5	-14.251456	-42.428266	-118.676256
	1.0	-14.072906	-42.097510	-118.176995
4	0.1	-17.746621	-73.081906	-284.990708
	0.3	-17.694721	-72.997332	-284.875402
	0.5	-17.634777	-72.894104	-284.728390
	1.0	-17.456227	-72.563347	-284.229129

In table 2, the numerical values of the skin-friction are computed for Pr=071, R=2, a=0.2 and Sc=0.16. It is observed that skin-friction increases with increasing thermal Grashof number or mass Grashof number.

Table 2
Numerical Values of skin-friction when Pr=0.71, Sc=0.16, R=2, a=0.2

Gr	Gc/t	0.1	0.2	0.3
2	2	-12.504385	-26.441876	-52.966821
5	5	-34.006653	-68.105763	-134.105860
15	10	-105.836638	-207.425935	-405.378661

Table 3 shows the numerical values of Nusselt number when Pr=0.71. From that it is clear that rate of heat transfer increases with increasing radiation parameter but the trend is reserved with respect to time t.

Table 3
Numerical Values of Nusselt Number when Pr=0.71

t /R	2	5	7	9
0.2	1.218862	2.240099	2.733876	3.139591
0.3	1.400831	2.407402	2.866427	3.242928
0.4	1.526350	2.497340	2.933246	3.296315
0.6	1.689142	2.596148	3.012449	3.367951

Table 4 shows the numerical values of Sherwood number. It is understood that the rate of mass transfer increases with increasing Schmidt number and time t.

Table 4
Numerical Values of Sherwood Number

Sc/t	0.2	0.3	0.4	0.6
0.16	0.201851	0.247215	0.285460	0.349615
0.3	0.276395	0.338514	0.390882	0.478731
0.6	0.390882	0.478731	0.552791	0.677028
2.01	0.715432	0.876221	1.011773	1.239164

CONCLUDING REMARKS

An analysis is performed to study the skin-friction, rate of heat transfer and rate of mass transfer effects on flow past an exponentially accelerated vertical plate with uniform temperate and variable mass diffusion in the presence of thermal radiation. Numerical computations are carried out for different physical parameters. The conclusions of the study are as follows:

(i) Skin-friction decreases with increasing radiation parameter and time t but the trend is just reserved with respect to exponential parameter, thermal Grashof number and mass Grashof number.

(ii) The rate of heat transfer increases with increasing radiation parameter and decreases with increasing time t.

(iii) The rate of mass transfer increases with increasing Schmidt number and time.

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NOMENCLATURE

A Constants
C' species concentration in the fluid $kg\,m^{-3}$
C dimensionless concentration
C_p specific heat at constant pressure $J.kg^{-1}.k$
D mass diffusion coefficient $m^2.s^{-1}$
Gc mass Grashof number
Gr thermal Grashof number

ISSN 2229-5518

g acceleration due to gravity $m.s^{-2}$
k thermal conductivity $W.m^{-1}.K^{-1}$
Pr Prandtl number
Sc Schmidt number
T temperature of the fluid near the plate K
t' time s
u velocity of the fluid in the x'-direction $m.s^{-1}$
u_0 velocity of the plate $m.s^{-1}$
u dimensionless velocity
y coordinate axis normal to the plate m
Y dimensionless coordinate axis normal to the plate

Greek symbols

β volumetric coefficient of thermal expansion K^{-1}
β^* volumetric coefficient of expansion with concentration K^{-1}
μ coefficient of viscosity $Ra.s$
ν kinematic viscosity $m^2.s^{-1}$
ρ density of the fluid $kg.m^{-3}$
τ dimensionless skin-friction $kg.m^{-1}.s^2$
θ dimensionless temperature
η similarity parameter
$erfc$ complementary error function

Subscripts

w conditions at the wall
∞ free stream conditions